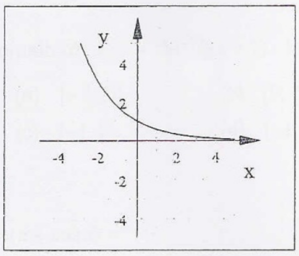
Kingdom of Saudi Arabia
King AbdulAziz University
Faculty of Science
Dept. of Mathematics

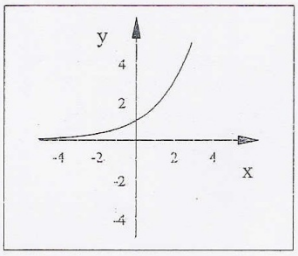
Instructions: (40 points) Solve each of the following problems and choose the correct answer :

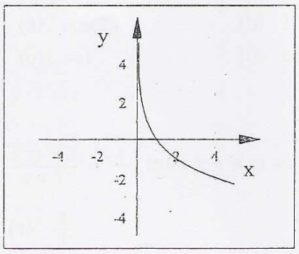
1. The domain of the logarithmic function $f(x) = \log_a(x)$ for any positive constant base a is

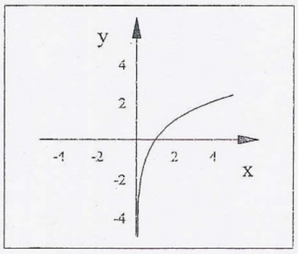
   (a) $(-\infty, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$
   (c) $(0, \infty)$ (d) $[0, \infty)$

2. The graph that represents the function $f(x) = \left(\frac{1}{3}\right)^x$ is

(a)


(b)


(c)


(d)


3. If $f(x) = \sqrt{x}$ and $g(x) = \sin(x)$, then the composition function $(f \circ g)(x) =$

(a) $\sqrt{\sin x}$ (b) $\sqrt{x \sin x}$

(c) $\sin x$ (d) $\sin(\sqrt{x})$

4. If the function $g(x) = f(x) - 1$ and the range of the function $f(x)$ is $[-2,2]$, then the range of $g(x)$ is

(a) $[-3,3]$ (b) $[-1,1]$

(c) $[-1,3]$ (d) $[-3,1]$

5. If $e^{x^3} = e^{27}$, then

(a) $x = -3$ (b) $x = 3$

(c) $x = e^3$ (d) $x = 9$

6. The domain of $f(x) = \sin^{-1}(3x+1)$ is

(a) $[-\frac{2}{3}, 0]$ (b) $[0, \frac{2}{3}]$

(c) $[-1,1]$ (d) $[-1, \frac{1}{3}]$

7. $\lim_{x\to 0} \cos(x + \cos x) =$

(a) $\cos(2)$ (b) 1

(c) -1 (d) $\cos(1)$

8. If $\lim_{x\to 1} \frac{f(x)+2}{x+3} = \frac{1}{3}$, then $\lim_{x\to 1} f(x) =$

(a) $\frac{4}{3}$ (b) $\frac{-2}{3}$

(c) $\frac{10}{3}$ (d) $\frac{-4}{3}$

9. The function $g(s) = \frac{s^3+27}{s+3}$ has a removable discontinuity at

(a) $s = 3$ (b) $s = -27$
(c) $s = -3$ (d) $s = 0$

10. $\lim_{t\to\infty} \frac{7t^3 + t^2}{(1-t)(5t^2-2)} =$

(a) $\frac{1}{5}$ (b) $\frac{-7}{5}$
(c) $\frac{7}{2}$ (d) $\frac{-7}{2}$

11. If the expression $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h}$ represents the derivative of a function $f(x)$ at a number a, then

(a) $f(x) = x^{10} + x, \quad a = 1$ (b) $f(x) = x^{10}, a = 0$
(c) $f(x) = x^{10} + x, \quad a = 0$ (d) $f(x) = x^{10}, a = 1$

12. The x-coordinate of the points on the curve $y = x^3 - 2x^2 - 5$ where the tangent line is horizontal are

(a) $x_1 = \frac{4}{3}, x_2 = -\frac{4}{3}$ (b) $x_1 = 0, x_2 = -\frac{4}{3}$
(c) $x_1 = 0, x_2 = \frac{4}{3}$ (d) $x_1 = 0, x_2 = \frac{-1}{3}$

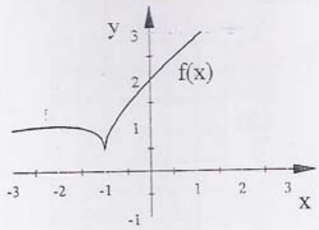
13. The function $f(x) = |x-1|$ is continuous and differentiable at $a = 1$

(a) True (b) False

14. The slope of the normal line to the curve $y = x^4 + 2e^x$ at the point $(0, 2)$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) -2 (d) 2

15. The graph of $f(x)$ in the accompanying figure shows that $f(x)$ is not differentiable at $x = -1$



(a) True (b) False

16. If $f(x) = -x^4 + 2x^3 - 10$, then $f^{(9)}(x) =$

(a) 0 (b) $24x$

(c) $-24x$ (d) -24

17. If $y = 5\ln\pi$, then $y' =$

(a) $-5\pi^{-1}$ (b) $5\pi^{-1}$

(c) 5π (d) 0

18. If $y = \frac{x^{10} - x^{-2}}{x}$, then $y' =$

(a) $9x^8 - 3x^{-4}$ (b) $9x^8 + x^{-3}$

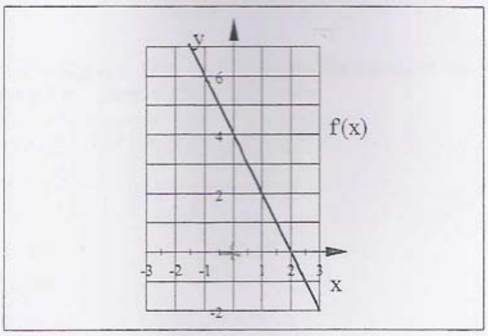
(c) $9x^8 + 3x^{-4}$ (d) $9x^8 - x^{-3}$

19. If $f(x) = \frac{e^x}{\cos\pi}$, then $f^{(10)}(x) =$

(a) $\frac{e^x}{(\cos\pi)^2}$ (b) $\frac{e^x}{\cos\pi}$

(c) $\left(\frac{e^x}{\cos\pi}\right)^{10}$ (d) $\frac{-e^x\sin\pi}{(\cos\pi)^2}$

20. If $f'(x)$ is the function whose graph is shown, and if $G(x) = \frac{-f(x)}{3}$, then $G'(1) =$



(a) $-\frac{2}{3}$ (b) $-\frac{1}{3}$

(c) 0 (d) $-\infty$

21. If $f(x) = \sin x$, then $f^{(29)}(x) =$

(a) $-\cos x$ (b) $\cos x$

(c) $\sin x$ (d) $-\sin x$

22. If $f(x) = x^4 \tan(e^x)$, then $f'(x) =$

(a) $4x^3\tan(e^x) - x^4e^x\sec^2(e^x)$ (b) $4x^3\tan(e^x) - x^4e^x\sec(e^x)\tan(e^x)$

(c) $x^4e^x\sec^2(e^x) + 4x^3\tan(e^x)$ (d) $4x^3\tan(e^x) + x^4e^x\sec(e^x)\tan(e^x)$

23. $\lim\limits_{t\to 0^+} t\csc t =$

(a) 1 (b) -1

(c) 0 (d) ∞

24. If $h(x) = (3rx^5)^s$ where r, s are constants, then $h'(x) =$

(a) $s(15rx^4)^{s-1}$ (b) $s(3rx^5)^{s-1}$

(c) $15r(3rx^5)^s$ (d) $15srx^4(3rx^5)^{s-1}$

25. If $h(x) = f(g(x))$, find $h'(0)$ using the values of the functions $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$ given in the table below

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	3	1	4	1
1	1	8	5	7

(a) 4 (b) 5

(c) 1 (d) 3

26. If $3x^2 - 2y^3 = 8$, then $\frac{d^2y}{dx^2} =$

(a) $-\frac{1}{y^2} - 2\frac{x^2}{y^5}$ (b) $\frac{1}{y^2} + 2\frac{x^2}{y^5}$

(c) $2\frac{x^2}{y^5} - \frac{1}{y^2}$ (d) $\frac{1}{y^2} - 2\frac{x^2}{y^5}$

27. If $\frac{1}{y} - \frac{1}{x} = 1$, then $y' =$

(a) x^2 (b) $\frac{-x^2}{y^2}$

(c) $\frac{y^2}{x^2}$ (d) x^2y^2

28. If $y = \tan^{-1}(\ln x)$, then $\frac{dy}{dx} =$

(a) $\frac{1}{x(1+(\ln x)^2)}$ (b) $\frac{x}{(1+(\ln x)^2)}$

(c) $\frac{-x}{(1+(\ln x)^2)}$ (d) $\frac{-1}{x(1+(\ln x)^2)}$

29. The tangent line to the curve of the function $y - 1 = \ln(xy^4)$ at the point $(1,1)$ is

(a) $3y + x + 4 = 0$ (b) $3y - x - 4 = 0$

(c) $3y + x - 4 = 0$ (d) $3y - x + 4 = 0$

30. If $y = (\sin x)^x$, then $y' =$

(a) $\ln \sin x + x\cot x$ (b) $(\sin x)^x(\ln \sin x + x\cot x)$
(c) $(\sin x)^x(\ln \sin x - x\cot x)$ (d) $(\sin x)^x(\sin x + x\cos x)$

31. The function $f(x) = x^3 + e^x$ is increasing $\forall x \in$

(a) $(-\infty, 0)$ (b) $(0, \infty)$
(c) $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ (d) $(-\infty, \infty)$

32. The critical numbers of the function $f(x) = \frac{x-1}{x^2+8}$ are:

(a) $-2, 4$ (b) $-1, -8$
(c) $2, 4$ (d) $1, 8$

33. If $f(x) = x^3 - 9x$, $x \in [0,3]$, then the number c that satisfy the conclusion of Roll's theorem is

(a) $-\sqrt{3}$ (b) 3
(c) $\sqrt{3}$ (d) -3

34. If $f'(x) = 0, \forall x \in (-1, 7)$, then the function $f(x)$ is

(a) Cubic on $(-1, 7)$ (b) Constant on $(-1, 7)$
(c) Linear on $(-1, 7)$ (d) Quadratic on $(-1, 7)$

35. The local minimum value of the function $g(x) = 6 - 3x + x^3$ is

(a) 4 (b) 8
(c) 1 (d) -1

36. The inflection points of the curve $y = x^3 - 12x + 1$ are

(a) $(0,0)$ (b) $(0,1)$
(c) $(-2,17)$ (d) $(2,-15)$

37. $\lim_{x \to 0^+} (\csc x - \frac{1}{x}) =$

(a) -1 (b) $-\infty$
(c) 0 (d) ∞

38. $\lim_{x \to \infty} \frac{\ln\sqrt{x}}{x^2} =$

(a) 0 (b) ∞
(c) 1 (d) $-\infty$

39. $\lim_{x \to \frac{\pi}{2}^-} (1 - \cos 3x)^{\sec x} =$

(a) 3 (b) e^{-3}
(c) 0 (d) e^3

40. $\lim_{x \to 0} \frac{x}{\sin^{-1} x} =$

(a) 1 (b) -1
(c) 0 (d) ∞